

# MATHEMATICS

## Chapter 2: Linear Equations in One Variable



## Linear Equations in One Variable

1. An equation is a statement of equality of two algebraic expressions involving one or more unknown quantities.

2. An equation involving only a linear polynomial is called a linear equation.

For example:  $\frac{2x}{5} - 4 = \frac{1}{2}, \frac{3t}{2} + \frac{t-7}{3} = 11.$

3. Any value of the variable which makes the equation a true statement is called the solution or root of the equation.

For example: - 2 is root of the equation  $3x - 2 = -8.$

4. Any term of an equation may be taken to the other side with its sign changed, without affecting the equality. This process is called transposition.

5. Without changing the equality, we may

- i. add the same quantity to both sides of the equation.
- ii. subtract the same quantity from both sides of the equation.
- iii. multiply both sides of the equation by the same non-zero quantity.
- iv. divide both sides of the equation by the same non-zero quantity.

6. If  $\frac{ax+b}{cx+d} = \frac{p}{q}$ , then  $q(ax + b) = p(cx + d)$

This process is called cross multiplication.

### Solving Linear Equations

#### Performing Mathematical Operations on Equations

When we are doing mathematical operations on a linear equation, we should do it on both sides of the equality otherwise the equality won't hold true.

Suppose,  $4x + 3 = 3x + 7$  is a linear equation. If we want to subtract 3 from the given equation, then we do it on both sides of the equality, so that the equality holds true.

$$4x + 3 - 3 = 3x + 7 - 3$$

$$\Rightarrow 4x = 3x + 4$$

Similarly, if we want to multiply or divide the equation, we multiply or divide all the terms on the left side of the equality and to the right side of the equality by the given number.

### Linear Equations

There are six main methods to solve linear equations. These methods for finding the solution of linear equations are:

Graphical Method

Elimination Method

Substitution Method

Cross Multiplication Method

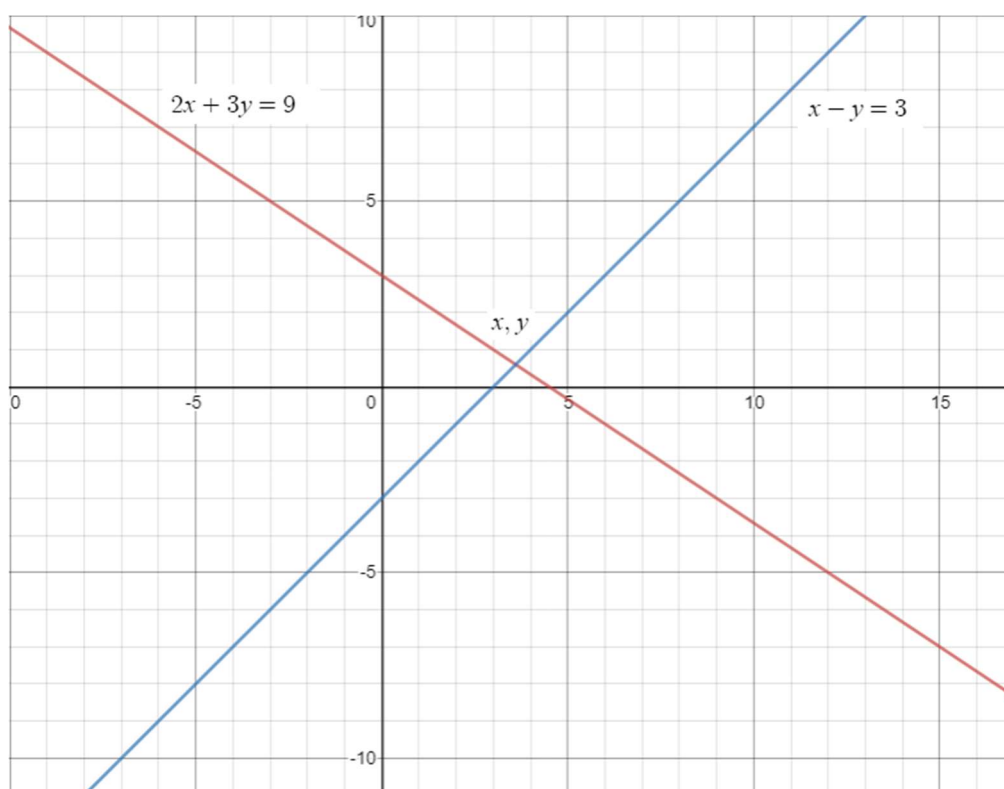
Matrix Method

Determinants Method

### Graphical Method of Solving Linear Equations

To solve linear equations graphically, first graph both equations in the same coordinate system and check for the intersection point in the graph. For example, take two equations as  $2x + 3y = 9$  and  $x - y = 3$ .

Now, to plot the graph, consider  $x = \{0, 1, 2, 3, 4\}$  and solve for  $y$ . Once  $(x, y)$  is obtained, plot the points on the graph. It should be noted that by having more values of  $x$  and  $y$  will make the graph more accurate.



In the graph, check for the intersection point of both the lines. Here, it is mentioned as  $(x, y)$ . Check the value of that point and that will be the solution of both the given equations. Here, the value of  $(x, y) = (3.6, 0.6)$ .

### Elimination Method of Solving Linear Equations

In the elimination method, any of the coefficients is first equated and eliminated. After elimination, the equations are solved to obtain the other equation. Below is an example of solving linear equations using the elimination method for better understanding.

Consider the same equations as

$$2x + 3y = 9 \text{ ———(i)}$$

And,

$$x - y = 3 \text{ ———(ii)}$$

Here, if equation (ii) is multiplied by 2, the coefficient of “ $x$ ” will become the same and can

be subtracted.

So, multiply equation (ii)  $\times 2$  and then subtract equation (i)

$$2x + 3y = 9$$

(-)

$$2x - 2y = 6$$

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$$-5y = -3$$

$$\text{Or, } y = \frac{3}{5} = 0.6$$

Now, put the value of  $y = 0.6$  in equation (ii).

$$\text{So, } x - 0.6 = 3$$

$$\text{Thus, } x = 3.6$$

In this way, the value of  $x, y$  is found to be 3.6 and 0.6.

### Substitution Method of Solving Linear Equations

To solve a linear equation using the substitution method, first, isolate the value of one variable from any of the equations. Then, substitute the value of the isolated variable in the second equation and solve it. Take the same equations again for example.

Consider,

$$2x + 3y = 9 \text{ ————(i)}$$

And,

$$x - y = 3 \text{ ————(ii)}$$

Now, consider equation (ii) and isolate the variable “ $x$ ”.

So, equation (ii) becomes,

$$x = 3 + y.$$

Now, substitute the value of  $x$  in equation (i). So, equation (i) will be-

$$2x + 3y = 9$$

$$\Rightarrow 2(3 + y) + 3y = 9$$

$$\Rightarrow 6 + 2y + 3y = 9$$

$$\text{Or, } y = \frac{3}{5} = 0.6$$

Now, substitute “ $y$ ” value in equation (ii).

$$x - y = 3$$

$$\Rightarrow x = 3 + 0.6$$

$$\text{Or, } x = 3.6$$

Thus,  $(x, y) = (3.6, 0.6)$ .

### Cross Multiplication Method of Solving Linear Equations

Linear equations can be easily solved using the cross multiplication method. In this method, the cross-multiplication technique is used to simplify the solution. For the cross-multiplication method for solving 2 variable equation, the formula used is:

$$x / (b_1 c_2 - b_2 c_1) = y / (c_1 a_2 - c_2 a_1) = 1 / (b_2 a_1 - b_1 a_2)$$

For example, consider the equations

$$2x + 3y = 9 \text{ ———(i)}$$

And,

$$x - y = 3 \text{ ———(ii)}$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -9$$

$$a_2 = 1, b_2 = -1, c_2 = -3$$

Now, solve using the afore mentioned formula.

$$x = (b_1 c_2 - b_2 c_1) / (b_2 a_1 - b_1 a_2)$$

Putting the respective value we get,

$$x = 18/5 = 3.6$$

Similarly, solve for y.

$$y = (c_1 a_2 - c_2 a_1) / (b_2 a_1 - b_1 a_2)$$

$$\text{So, } y = \frac{3}{5} = 0.6$$

### Matrix Method of Solving Linear Equations

Linear equations can also be solved using matrix method. This method is extremely helpful for solving linear equations in two or three variables. Consider three equations as:

$$a_1x + a_2y + a_3z = d_1$$

$$b_1x + b_2y + b_3z = d_2$$

$$c_1x + c_2y + c_3z = d_3$$

These equations can be written as:

$$\begin{bmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\Rightarrow AX = B \text{ ———(i)}$$

Here, the A matrix, B matrix and X matrix are:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Now, multiply (i) by  $A^{-1}$  to get:

$$A^{-1}AX = A^{-1}B \Rightarrow I.X = A^{-1}B$$

$$\Rightarrow X = A^{-1}B$$

### Determinant Method of Solving Linear Equations (Cramer's Rule)

Determinant's method can be used to solve linear equations in two or three variables easily. For two variables and three variables of linear equations, the procedure is as follows.

#### For Linear Equations in Two Variables:

$$x = \Delta_1 / \Delta,$$

$$y = \Delta_2 / \Delta$$

$$\text{Or, } x = (b_1 c_2 - b_2 c_1) / (b_2 a_1 - b_1 a_2) \text{ and } y = (c_1 a_2 - c_2 a_1) / (b_2 a_1 - b_1 a_2)$$

Here,

$$\Delta_1 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, \Delta_2 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} \text{ and } \Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

#### For Linear Equations in Three Variables:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

**Note:** we cannot multiply or divide the equation by 0.

### Solving Equations with Linear Expression on one side and numbers on the other Side

Suppose we have to find the solution of  $2x-3=7$ , where the linear expression is on the left-hand side, and numbers on the right-hand side.

**Step 1:** Transpose all the constant terms from the left-hand side to the right-hand side.

$$2x = 7 + 3 = 10 \Rightarrow 2x = 10$$

**Step 2:** Divide both sides of the equation by the coefficient of the variable.

In the above equation  $2x$  is on the left-hand side. The coefficient of  $2x$  is 2.

On dividing the equation by two, We get:

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 10$$

$$\Rightarrow x = \frac{10}{2} = 5, \text{ Which is the required solution.}$$

Solving Equations with variables on both sides

Suppose we have to solve  $3x - 3 = x + 2$ . In this equation, there are variables on both sides of the equation.

**Step 1:** Transpose all the terms with a variable from the right-hand side to the left-hand side of the equation and all the constants from the left-hand side to the right-hand side of the equation.

$$3x - x = 2 + 3$$

$$\Rightarrow 2x = 5$$

**Step 2:** Divide both sides of the equation by the coefficient of the variable.

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 5$$

### Applications (Word Problems)

Sum of two numbers is 74. One of the numbers is 10 more than the other. What are the numbers?

Let one of the numbers be  $x$ .

Then the other number is  $x + 10$ .

Given that the sum of the two numbers is 74.

$$\text{So, } x + (x + 10) = 74$$

$$\Rightarrow 2x + 10 = 74$$

$$\Rightarrow 2x = 74 - 10 = 64$$

$$\Rightarrow x = \frac{64}{2} = 32$$

One of the numbers is 32 and the other number is 42.

### Equations Reducible to the Linear Form

$$\frac{x+1}{2x+3} = \frac{3}{8}$$

Multiplying both sides with  $2x + 3$

$$\Rightarrow \frac{x+1}{2x+3} \times (2x+3) = \frac{3}{8} \times (2x+3)$$

$$\Rightarrow x + 1 = \frac{3(2x+3)}{8}$$

Multiplying both sides with 8

$$\Rightarrow 8(x + 1) = 3(2x + 3)$$

$$\Rightarrow 8x + 8 = 6x + 9$$

$$\Rightarrow 8x = 6x + 9 - 8$$

$$\Rightarrow 8x = 6x + 1$$

$$\Rightarrow 8x - 6x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

### Reducing Equations to Simpler Form

Simplify the equation  $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$ .

$$\frac{6x+1}{3} + 1 = \frac{x-3}{6}$$

$$\Rightarrow \frac{6(6x+1)}{3} + 6 \times 1 = \frac{6(x-3)}{6} \quad (\text{Multiplying both sides by 6})$$

$$\Rightarrow 2(6x + 1) + 6 = (x - 3)$$

$$\Rightarrow 12x + 2 + 6 = x - 3 \quad (\text{opening the brackets})$$

$$\Rightarrow 12x + 8 = x - 3$$

$$\Rightarrow 12x - x + 8 = -3$$

$$\Rightarrow 11x + 8 = -3$$

$$\Rightarrow 11x = -3 - 8$$

$$\Rightarrow 11x = -11$$

$$\Rightarrow x = -1 \quad (\text{required solution})$$

$$\text{LHS: } \frac{6(-1)+1}{3} + 1 = \frac{-6+1}{3} + 1 = \frac{-5}{3} + \frac{3}{3} = \frac{-2}{3}$$

$$\text{RHS: } \frac{(-1)-3}{6} = \frac{-4}{6} = \frac{-2}{3}$$

$$\text{LHS} = \text{RHS}$$

## Introduction to Linear Equations in One Variable

### Variables and Constants

A constant is a value or number that never changes in an expression and it's constantly the same.

A variable is a letter representing some unknown value. Its value is not fixed, it can take any value. On the other hand, the value of a constant is fixed.

For example, in the expression  $4x+7$ , 4 and 7 are the constants and  $x$  is a variable.

### Algebraic Equation

The statement of equality of two algebraic expressions is an algebraic equation. It is of the form  $P=Q$ , where  $P$  and  $Q$  are algebraic expressions.

$6x + 5$  and  $5x + 3$  are algebraic expressions. On equating the algebraic expressions we get an algebraic equation.

$6x + 5 = 5x + 3$  is an algebraic equation.

### Linear Equations in One Variable

A linear equation is an algebraic equation in which each term is either a constant or the

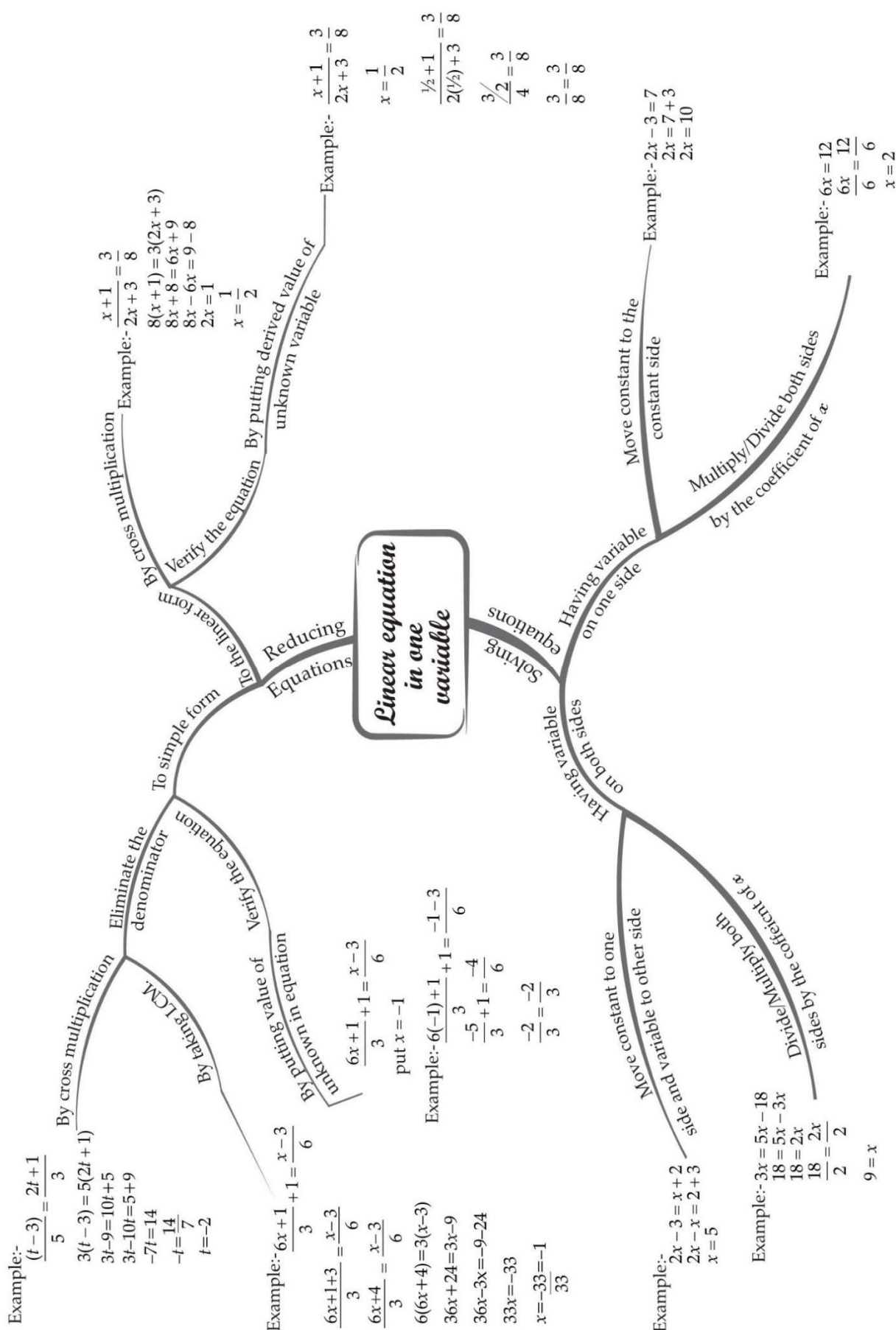
product of a constant and a single variable, where the highest power of the variable is one.

If the linear equation has only a single variable then it is called a linear equation in one variable.

For example,  $7x + 4 = 5x + 8$  is a linear equation in one variable.

# CHAPTER-4

## MIND MAP : LEARNING MADE SIMPLE



## Important Questions

### Multiple Choice Questions-

Question 1. Solve:  $3x = 12$

- (a) 15
- (b) 4
- (c) 9
- (d) 3

Question 2. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

- (a) 40 and 60
- (b) 50 and 55
- (c) 50 and 60
- (d) 40 and 55

Question 3. The sum of two-digit number and the number formed by interchanging its digit is 110. If ten is subtracted from the first number, the new number is 4 more than 5 times of the sum of the digits in the first number. Find the first number.

- (a) 46
- (b) 48
- (c) 64
- (d) 84

Question 4. Solve:  $7x = 21$

- (a) 3
- (b) 2
- (c) 14
- (d) none of these

Question 5. The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

- (a) 85
- (b) 58
- (c) 36
- (d) 76

Question 6. The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

- (a) 22 and 88
- (b) 44 and 66
- (c) 44 and 110
- (d) 33 and 99

Question 7. Solve:  $5x - 2(2x - 7) = (3x - 1) + \frac{7}{2}$

- (a) 2
- (b) 3
- (c) 12
- (d)  $\frac{23}{4}$

Question 8. Amina thinks of a number and subtracts  $\frac{5}{2}$  from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

- (a) 2
- (b) 3
- (c) 4
- (d) none of these

Question 9. Solve  $2x - 3 = x + 2$

- (a) 4
- (b) 5
- (c) 3
- (d) 0

Question 10. Find the solution of  $2x - 3 = 7$

- (a) 3
- (b) 4
- (c) 5
- (d) none of these

### Very Short Questions:

1. Identify the algebraic linear equations from the given expressions.

- (a)  $x^2 + x = 2$
- (b)  $3x + 5 = 11$
- (c)  $5 + 7 = 12$
- (d)  $x + y^2 = 3$

2. Check whether the linear equation  $3x + 5 = 11$  is true for  $x = 2$ .
3. Form a linear equation from the given statement: 'When 5 is added to twice a number, it gives 11.'
4. If  $x = a$ , then which of the following is not always true for an integer  $k$ .
  - (a)  $kx = ak$
  - (b)  $\frac{x}{k} = \frac{a}{k}$
  - (c)  $x - k = a - k$
  - (d)  $x + k = a + k$
5. Solve the following linear equations:
  - (a)  $4x + 5 = 9$
  - (b)  $x + \frac{3}{2} = 2x$
6. Solve the given equation  $3\frac{1}{x} \times 5\frac{1}{4} = 17\frac{1}{2}$
7. Verify that  $x = 2$  is the solution of the equation  $4.4x - 3.8 = 5$ .
- 8.

Solve  $\frac{3x}{4} - \frac{2x+5}{3} = \frac{5}{2}$

9. The angles of a triangle are in the ratio  $2 : 3 : 4$ . Find the angles of the triangle.
10. The sum of two numbers is 11 and their difference is 5. Find the numbers.

### Short Questions:

1. If the sum of two consecutive numbers is 11, find the numbers.
2. The breadth of a rectangular garden is  $\frac{2}{3}$  of its length. If its perimeter is 40 m, find its dimensions.
3. The difference between two positive numbers is 40 and the ratio of these integers is  $1 : 3$ . Find the integers.
4. Solve for  $x$ :

$$\frac{7x+14}{3} - \frac{17-3x}{5} = 6x - \frac{4x+2}{3} - 5$$

5. The sum of a two-digit number and the number obtained by reversing its digits is 121. Find the number if it's unit place digit is 5.

### Long Questions:

1. If the length of the rectangle is increased by 40% and its breadth is decreased by 40%, what will be the percentage change in its perimeter?

2. A fruit seller buys some oranges at the rate of ₹ 5 per orange. He also buys an equal number of bananas at the rate of ₹ 2 per banana. He makes a profit of 20% on oranges and a profit of 15% on bananas. In the end, he sold all the fruits. If he earned a profit of ₹ 390, find the number of oranges.
3. A steamer goes downstream from one point to another in 7 hours. It covers the same distance upstream in 8 hours. If the speed of stream be 2 km/h, find the speed of the steamer in still water and the distance between the ports.

### Answer Key-

#### Multiple Choice questions-

1. (b) 4
2. (d) 40 and 55
3. (c) 64
4. (a) 3
5. (a) 85
6. (c) 44 and 110
7. (d)  $\frac{23}{4}$
8. (c) 4
9. (b) 5
10. (c) 5

#### Very Short Answer:

1. (a)  $x^2 + x = 2$  is not a linear equation.  
 (b)  $3x + 5 = 11$  is a linear equation.  
 (c)  $5 + 7 = 12$  is not a linear equation as it does not contain variable.  
 (d)  $x + y^2 = 3$  is not a linear equation.
2. Given that  $3x + 5 = 11$   
 For  $x = 2$ , we get  

$$\text{LHS} = 3 \times 2 + 5 = 6 + 5 = 11$$

$$\text{LHS} = \text{RHS} = 11$$
 Hence, the given equation is true for  $x = 2$
3. As per the given statement we have  
 $2x + 5 = 11$  which is the required linear equation.
4. Correct answer is (b).
5. (a) We have  $4x + 5 = 9$

$$\Rightarrow 4x = 9 - 5 \text{ (Transposing 5 to RHS)}$$

$$\Rightarrow 4x = 4$$

$$\Rightarrow x = 1 \text{ (Transposing 4 to RHS)}$$

$$\text{(b) We have } x + \frac{3}{2} = 2x$$

$$\Rightarrow \frac{3}{2} = 2x - x$$

$$\Rightarrow x = \frac{3}{2}$$

$$6. \text{ We have } 3\frac{1}{x} \times 5\frac{1}{4} = 17\frac{1}{2}$$

$$\Rightarrow \frac{3x+1}{x} \times \frac{21}{4} = \frac{35}{2}$$

$$\Rightarrow \frac{3x+1}{x} = \frac{35}{2} \div \frac{21}{4}$$

$$\left( \text{Transposing } \frac{21}{4} \text{ to RHS} \right)$$

$$\Rightarrow \frac{3x+1}{4} = \frac{35}{2} \times \frac{4}{21}$$

$$\Rightarrow \frac{3x+1}{4} = \frac{10}{3}$$

$$\Rightarrow 3(3x+1) = 4 \times 10 \text{ (Cross-multiplication)}$$

$$\Rightarrow 9x+3 = 40 \text{ (Solving the brackets)}$$

$$\Rightarrow 9x = 40 - 3 \text{ (Transposing 3 to RHS)}$$

$$\Rightarrow 9x = 37$$

$$\therefore x = \frac{37}{9}$$

$$\text{Hence the required solution is } x = \frac{37}{9}$$

$$7. \text{ We have } 4.4x - 3.8 = 5$$

Putting  $x = 2$ , we have

$$4.4 \times 2 - 3.8 = 5$$

$$\Rightarrow 8.8 - 3.8 = 5$$

$$\Rightarrow 5 = 5$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Hence verified.

$$8.$$

We have  $\frac{3x}{4} - \frac{2x+5}{3} = \frac{5}{2}$

LCM of 2, 3 and 4 = 12

$$\therefore \frac{3x}{4} \times 12 - \frac{2x+5}{3} \times 12 = \frac{5}{2} \times 12$$

(Multiplying both sides by 12)

$$\Rightarrow 3x \times 3 - (2x + 5) \times 4 = 5 \times 6$$

$$\Rightarrow 9x - 8x - 20 = 30 \text{ (Solving the bracket)}$$

$$\Rightarrow x - 20 = 30$$

$$\Rightarrow x = 30 + 20 \text{ (Transposing 20 to RHS)}$$

$$\Rightarrow x = 50$$

Hence  $x = 50$  is the required solution.

9. Let the angles of a given triangle be  $2x^\circ$ ,  $3x^\circ$  and  $4x^\circ$ .

$$2x + 3x + 4x = 180 \text{ (}\because \text{Sum of the angles of a triangle is } 180^\circ\text{)}$$

$$\Rightarrow 9x = 180$$

$$\Rightarrow x = 20 \text{ (Transposing 9 to RHS)}$$

Angles of the given triangles are

$$2 \times 20 = 40^\circ$$

$$3 \times 20 = 60^\circ$$

$$4 \times 20 = 80^\circ$$

10. Let one of the two numbers be  $x$ .

$$\text{Other number} = 11 - x.$$

As per the conditions, we have

$$x - (11 - x) = 5$$

$$\Rightarrow x - 11 + x = 5 \text{ (Solving the bracket)}$$

$$\Rightarrow 2x - 11 = 5$$

$$\Rightarrow 2x = 5 + 11 \text{ (Transposing 11 to RHS)}$$

$$\Rightarrow 2x = 16$$

$$\Rightarrow x = 8$$

Hence the required numbers are 8 and  $11 - 8 = 3$

### Short Answer:

1. Let the two consecutive numbers be  $x$  and  $x + 1$ .

As per the conditions, we have

$$x + x + 1 = 11$$

$$\Rightarrow 2x + 1 = 11$$

$$\Rightarrow 2x = 11 - 1 \text{ (Transposing 1 to RHS)}$$

$$\Rightarrow 2x = 10$$

$$x = 5$$

Hence, the required numbers are 5 and  $5 + 1 = 6$ .

2. Let the length of the garden be  $x$  m

$$\text{its breadth} = \frac{2}{3} \times x \text{ m.}$$

$$\text{Perimeter} = 2 [\text{length} + \text{breadth}]$$

$$2 \left( x + \frac{2}{3}x \right) = 40$$

$$\Rightarrow 2x + \frac{4}{3}x = 40 \quad \text{(Solving the bracket)}$$

$$\Rightarrow 3 \times 2x + 3 \times \frac{4}{3}x = 3 \times 40$$

(Multiplying both sides by 3)

$$\Rightarrow 6x + 4x = 120$$

$$\Rightarrow 10x = 120$$

$$\therefore x = \frac{120}{10} = 12$$

$$\therefore \text{Length} = 12 \text{ m and breadth } \frac{2}{3}x = 12 \times \frac{2}{3} = 8 \text{ m.}$$

3. Let one integer be  $x$ .

$$\text{Other integer} = x - 40$$

As per the conditions, we have

$$\frac{x - 40}{x} = \frac{1}{3}$$

$$\Rightarrow 3(x - 40) = x$$

$$\Rightarrow 3x - 120 = x$$

$$\Rightarrow 3x - x = 120$$

$$\Rightarrow 2x = 120$$

$$\Rightarrow x = 60$$

Hence the integers are 60 and  $60 - 40 = 20$ .

- 4.

We have

$$\frac{7x+14}{3} - \frac{17-3x}{5} = 6x - \frac{4x+2}{3} - 5$$

LCM of 3 and 5 = 15

$$\begin{aligned} \frac{7x+14}{3} \times 15 - \frac{17-3x}{5} \times 15 &= \\ 6x \times 15 - \frac{4x+2}{3} \times 15 - 5 \times 15 & \end{aligned}$$

(Multiplying both sides by 15)

$$\begin{aligned} \Rightarrow (7x + 14) \times 5 - (17 - 3x) \times 3 &= \\ = 90x - (4x + 2) \times 5 - 75 & \end{aligned}$$

$$\begin{aligned} \Rightarrow 35x + 70 - 51 + 9x &= \\ = 90x - 20x - 10 - 75 & \end{aligned}$$

(Solving the brackets)

$$\Rightarrow 44x + 19 = 70x - 85$$

$$\Rightarrow 44x - 70x = -85 - 19$$

(Transposing 70x to LHS and 19 to RHS)

$$\Rightarrow -26x = -104$$

$$\therefore x = \frac{-104}{-26} = 4$$

Hence  $x = 4$  is the required number

5. Unit place digit is given as 5

Let x be the tens place digit

Number formed =  $5 + 10x$

Number obtained by reversing the digits =  $5 \times 10 + x = 50 + x$

As per the conditions, we have

$$5 + 10x + 50 + x = 121$$

$$\Rightarrow 11x + 55 = 121$$

$$\Rightarrow 11x = 121 - 55 \text{ (Transposing 55 to RHS)}$$

$$\Rightarrow 11x = 66$$

$$\Rightarrow x = 6$$

Thus, the tens place digit = 6

Hence the required number =  $5 + 6 \times 10 = 5 + 60 = 65$

## Long Answer:

- Let the length of the rectangle be  $x$  m and its breadth be  $y$  m

$$\text{Perimeter} = 2(x + y)$$

Now the length of the rectangle becomes after a 40% increase

$$= x + \frac{40}{100}x = \frac{140}{100}x = \frac{7}{5}x$$

Breadth of the rectangle becomes after 40% decrease

$$= y - \frac{40}{100}y = \frac{60}{100}y = \frac{3}{5}y$$

$$\text{New perimeter} = 2\left[\frac{7x}{5} + \frac{3y}{5}\right] \text{ m}$$

$\therefore$  Change in perimeter

$$= 2\left[\frac{7x}{5} + \frac{3y}{5}\right] - 2[x + y]$$

$$= \frac{14x}{5} + \frac{6y}{5} - 2x - 2y = \frac{14x}{5} - 2x + \frac{6y}{5} - 2y$$

$$= \left(\frac{4x}{5} - \frac{4y}{5}\right) \text{ m}$$

$$\text{Percentage of change} = \frac{\frac{4x}{5} - \frac{4y}{5}}{2(x + y)} \times 100$$

$$= \frac{\frac{4}{5}(x - y)}{2(x + y)} \times 100 = \frac{4}{5 \times 2} \left(\frac{x - y}{x + y}\right) \times 100$$

$$= 40 \left(\frac{x - y}{x + y}\right) \%$$

Hence the required change percentage

$$= 40 \left(\frac{x - y}{x + y}\right) \%$$

- Let the number of oranges bought by him be  $x$  and also the number of bananas be  $x$ .

Cost of  $x$  oranges at the rate of ₹ 5 per orange = ₹  $5x$

Cost of  $x$  bananas at the rate of ₹ 2 per banana = ₹  $2x$

$$\text{Profit earned on oranges} = \frac{20}{100} \times 5x = ₹ x$$

$$\text{Profit earned on bananas} = \frac{15}{100} \times 2x = ₹ \frac{3}{10}x$$

As per the conditions, we have

$$x + \frac{3}{10}x = 390$$

$$\Rightarrow \frac{10x + 3x}{10} = 390$$

$$\Rightarrow \frac{13x}{10} = 390$$

$$\Rightarrow x = 390 \times \frac{10}{13}$$

$$\left( \text{Transposing } \frac{13}{10} \text{ to RHS} \right)$$

$$\Rightarrow x = 30 \times 10 = 300$$

Hence, the number of oranges = 300.

3. Let speed of steamer in still water =  $x$  km/h

Speed of stream = 2 km/h

Speed downstream =  $(x + 2)$  km/h

Speed upstream =  $(x - 2)$  km/h

Distance covered in 7 hours while downstream =  $7(x + 2)$

Distance covered in 8 hours while upstream =  $8(x - 2)$

According to the condition,

$$7(x + 2) = 8(x - 2)$$

$$\Rightarrow 7x + 14 = 8x - 16$$

$$\Rightarrow x = 30 \text{ km/h}$$

$$\text{Total Distance} = 7(x + 2) \text{ km} = 7(30 + 2) \text{ km} = 7 \times 32 \text{ km} = 224 \text{ km.}$$